Bull Volcanol (1994) 56:417–424

ORIGINAL PAPER

P. T. Delaney · D. F. McTigue

Volume of magma accumulation or withdrawal estimated from surface uplift or subsidence, with application to the 1960 collapse of Kilauea Volcano

Received February 28, 1994 / Accepted August 10, 1994

Abstract An elastic point source model proposed by Mogi for magma chamber inflation and deflation has been applied to geodetic data collected at many volcanoes. The volume of ground surface uplift or subsidence estimated from this model is closely related to the volume of magma injection into or withdrawal from the reservoir below. The analytical expressions for these volumes are reviewed for a spherical chamber and it is shown that they differ by the factor $2(1-\nu)$, where ν is Poisson's ratio of the host rock. For the common estimate $\nu=0.25$, as used by Mogi and subsequent workers, the uplift volume is 3/2 the injection volume. For highly fractured rocks, ν can be even less and the uplift volume can approach twice the injection volume. Unfortunately, there is no single relation between the inflation of magma reservoirs and the dilation or contraction of host rocks. The inflation of sill-like bodies, for instance, generates no overall change in host rock volume. Inflation of dike-like bodies generates contraction such that, in contrast with Mogi's result, the uplift volume is generally less than the injection volume; for $\nu=0.25$, the former is only 3/4 of the latter. Estimates of volumes of magma injection or withdrawal are therefore greatly dependent on the magma reservoir configuration. Ground surface tilt data collected during the 1960 collapse of Kilauea crater, one of the first events interpreted with Mogi's model and one of the largest collapses measured at Kilauea, is not favored by any one of a variety of deformation models. These models, however, predict substantially different volumes of both magma withdrawal and ground surface subsidence.

P. T. Delaney (✉)
US Geological Survey, 2255 North Gemini Drive,
Flagstaff, AZ 86001, USA
e-mail: delaney@aa.wr.usgs.gov

D. F. McTigue
Department of Geological Sciences, University of Washington,
Seattle, WA 98195, USA
e-mail: mctigue@u.washington.edu

Key words Mogi · dislocations · deformation Kilauea

Introduction

A survey of applications of the Mogi model to the interpretation of geodetic data reveals some inconsistencies in the estimation of the volume of magma accumulation or withdrawal. Some workers equate the volume of ground surface uplift with the volume of magma accumulation below. These two volumes are not, in general, equal. The volume of uplift is the sum of the volume of magma reservoir expansion and the dilation or contraction of the country rocks in response to that expansion. Even where data for elevation change collected across a volcano are sufficiently complete to allow model-independent calculation of the volume of uplift or subsidence (Jackson et al. 1975), the volume of magma accumulation or withdrawal remains uncertain for the same reason.

The Mogi model idealizes deformation caused by magma injection as a spherically symmetrical point source of expansion or contraction. Ground surface deformation data can be fitted to obtain the position and strength of the source. The model cannot, however, separate the size of the chamber from the magnitude of the internal pressure change, thus obscuring their relation to the volume of magma accumulation or withdrawal responsible for the deformation. In his original work, Mogi (1958) tabulated volumes of ground surface uplift or subsidence calculated from estimates of the model parameters obtained for certain periods at various volcanoes. Although this tabulation implies an interest in volumes of magma transfer, he offered no conclusions regarding the behavior of the reservoirs responsible for the ground surface motions.

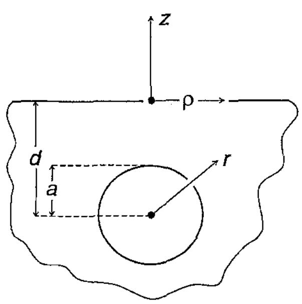
Mogi's model was soon used to interpret ground surface tilt data collected at Kilauea volcano (Eaton 1962), where the volume of lava erupted along the lower east rift zone in early 1960 approximately agreed with the volume of calculated ground surface subsidence at the summit 50–60 km away. This equality seems to have been accepted as generally valid in published work on Hawaiian volcanism, although only Dvorak et al. (1983) explicitly mention and use Eaton's estimates as justification. In contrast, for instance, magma accumulation beneath Long Valley caldera (Savage and Clark 1982; Rundle and Whitcomb 1984) was examined by fitting data to Mogi's model and then using an expression for the so-called injection volume, equivalent to that derived in this paper. This expression is found neither in Mogi's (1958) paper nor those on Long Valley caldera. The expression requires that an elastic constant, Poisson's ratio, is specified. For the value used by Mogi and all subsequent workers, the volume of uplift or subsidence is 50% greater than the volume of magma injected into or withdrawn from the underlying chamber. The subsidence volume of the 1960 Kilauea summit collapse determined by application of Mogi's model was $150 \times 10^6\,m^3$ (Eaton 1962). Although this figure is in good agreement with the $120 \times 10^6\,m^3$ of lava erupted, the withdrawal volume calculated from the same model parameters was $100 \times 10^6\,m^3$ and agrees equally well.

Mogi's model is not the only model that might be considered when interpreting ground surface deformation data. We also present an expression for the ratio of uplift volume to injection volume appropriate for sheet-like reservoirs, idealized as infinitesimal planar sources and finite rectangular sources of uniform opening or closing. We then apply these models to the data collected by Eaton (1962) and discuss the resultant uncertainties in the determination of injection volumes.

Spherical chamber

McTigue (1987) studied the deformation caused by the uniform pressurization of a spherical chamber in an elastic half-space (see Fig. 1). Solutions were cast as series expansions in increasing powers of the ratio of chamber radius a to its depth to center d such that results are exact as $a/d \to 0$. Importantly, whereas the first approximation of the normalized surface deformation is proportional to $(a/d)^2$, the next term is of the order $(a/d)^5$. The relatively simple first-order expressions used here therefore provide an accurate portrayal of ground surface motion even where a/d is as great as ≈ 0.5.

Fig. 1 Definition sketch for the expansion or contraction of a spherical magma chamber



The radial displacement of the chamber is [McTigue 1987, equation (11)]

$$u_r(a) = \frac{1}{4}\frac{\Delta P}{G}a \tag{1}$$

where ΔP is the magma pressure change and G is the elastic shear modulus. The uplift, or vertical component of the total displacement, at the free surface due to the chamber expansion is, in the first approximation

$$u_z(\rho) = u_z^{max}[1 + (\rho/d)^2]^{-3/2} \tag{2}$$

where

$$u_z^{max} = (1-\nu)\frac{\Delta P}{G}a(a/d)^2 \tag{3}$$

and where ρ and z are cylindrical polar coordinates with the origin on the free surface centered over the chamber and z positive upward (Fig. 1), ν is Poisson's ratio, and $u_z^{max} = u_z(0)$, the maximum surface uplift, is the vertical displacement at $\rho = 0$ [McTigue 1987, Equation (14)]. Chamber size, pressure change and the elastic constants are inseparable in the sense that many combinations of these parameters can produce the same surface displacement [Eq. (3)]. Equations (2) and (3) reduce to Mogi's (1958) result in the special case $\nu = 0.25$. Mogi presented a solution for a uniform point source of expansion without formal recognition that it is an excellent approximation for the motions caused by chambers of finite radius. Although more exact expressions that isolate chamber size are available (McTigue 1987), the influence of this effect is typically too weak to be extracted by parameter fits to data.

Integration of Equation (2) over the free surface yields the volume of uplift

$$\Delta V_{uplift} = 2\pi u_z^{max} d^2 \tag{4}$$

Noting that Mogi's model contains only a single length scale, the source depth d, it is not surprising that the uplift volume, as well as the injection volume derived below, is sensitive to it. Alternatively, we could choose to use Eq. (3) to eliminate d from Eq. (4) in favor of the chamber radius a, pressure change ΔP and shear modulus G. We emphasize that this stategy can be fruitful only if there exists a method to specify independently all but one of these parameters.

We now derive the volume change of the chamber caused by the pressurization. The volume of an undeformed spherical chamber is, of course, $V_o = 4\pi a^3/3$. That of the deformed chamber is

$$V_d = \frac{4}{3}\pi[a + u_r(a)]^3 \simeq \frac{4}{3}\pi a^3\left[1 + 3\frac{u_r(a)}{a} + \cdots\right] \tag{5}$$

Neglect of higher order terms in the series expansion is justified because displacements of the wall are small compared with the chamber radius, $u_r(a)/a \ll 1$. The volume change, or injection volume, responsible for the displacement of the chamber walls is then given by

$$\Delta V_{\text{injection}} = V_d - V_o = 4\pi a^2 u_r(a) \quad (6)$$

Insertion of Eqs. (1) and (3) into Eq. (6) yields expressions for the injection volume in terms of the pressure change or maximum vertical surface displacement

$$\Delta V_{\text{injection}} = \pi \frac{\Delta P}{G} a^3 = \frac{\pi u_z^{\max} d^2}{(1-\nu)} \quad (7)$$

As $a^3 \to 0$, it is necessary that $\Delta P/G \to \infty$ to maintain a finite strength of the point source of expansion, prohibiting the calculation of injection volume directly from the source parameters.

The ratio of the uplift volume to the injection volume depends only on Poisson's ratio; from Eqs. (4) and (7), we have

$$\frac{\Delta V_{\text{uplift}}}{\Delta V_{\text{injection}}} = 2(1-\nu) \quad (8)$$

For incompressible materials ($\nu = 0.5$), Eq. (8) indicates that the uplift and injection volumes are equal. However, compressibility ($\nu < 0.5$) increases the uplift volume relative to the injection volume and, from Eq. (3), surface uplift increases as Poisson's ratio decreases. For the often-assumed case of $\nu = 0.25$, consistent with a range of ≈ 0.15–0.30 obtained from laboratory measurements of intact rocks (Touloukian et al. 1981, Ch. 6), the uplift volume is 3/2 of the injection volume.

Insight into the reason for host rock dilation during uplift can be obtained from the results presented by McTigue (1987). Uniform expansion of a spherical chamber in an infinite elastic domain gives rise to no mean, or dilational, strain; radial contractional strains are everywhere exactly balanced by tangential extensional strains. To include a stress-free surface, tractions are applied at that surface to counter those due to the expansion in the unbounded domain. These tractions strain the half-space. At the order of approximation employed by Mogi (1958), the volumetric strain is

$$\varepsilon = (1-2\nu) \frac{\Delta P}{G} \left(\frac{a}{d}\right)^3 \frac{2(1-z/d)^2 - (\rho/d)^2}{[(1-z/d)^2 + (\rho/d)^2]^{5/2}} \quad (9)$$

[McTigue 1987, Eqs. (18)–(20)], where positive strain is extensional and z is positive upward from the free surface. Equation (9) represents a bowl-shaped zone of dilation above the magma chamber, with the maximum effect near the free surface, as shown in Fig. 2. Note from Eq. (9) that volume strain scales with $(1-2\nu)$, so that this contribution to the surface uplift vanishes for incompressible rock ($\nu \to 0.5$). Integration of Equation (9) over the entire half-space yields a total volume of dilation of $2\pi(1-2\nu)a^3 \Delta P/G$.

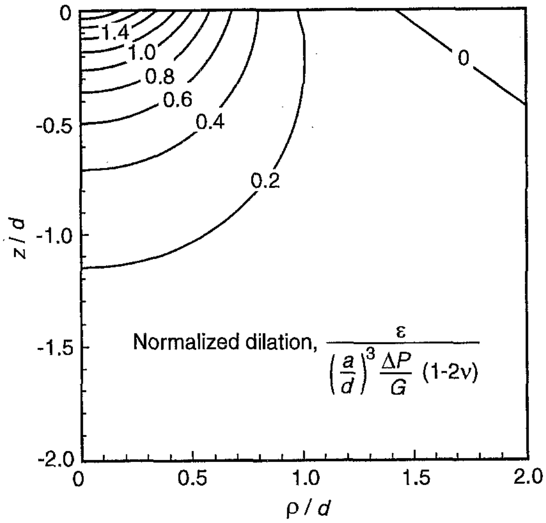


Fig. 2 Dilation associated with a point source of expansion

Sheet-like chambers

The point source of uniform expansion is a simplistic approximation and can be regarded as a 'far-field' model in that there exists only one source parameter, the dilation. A slightly more complex far-field model incorporates an infinitesimally small plane of directed opening, so that two orientation parameters, a strike and a dip, are included among the source characteristics. This solution has been presented by Okada (1985), among others, and is a simplistic model for inflation and deflation of sheet-like magma reservoirs that are infinitesimal in the sense that the dimensions of the sheet are small compared with its depth. Integration of the surface uplift [Okada's Equation (10)] yields the result

$$\frac{\Delta V_{\text{uplift}}}{\Delta V_{\text{injection}}} = 1 - \frac{(1-2\nu)}{2} \sin^2 \delta \quad (10)$$

where δ is the dip of the sheet such that $\delta = 0°$ and $\delta = 90°$ correspond to a sill and a dike, respectively. As with the spherical chamber, uplift and injection volumes are equal only for incompressible rock and the ratio of uplift volume to injection volume is independent of chamber depth d. Volumes of inflation and deflation of sills exactly equal the volumes of surface uplift and subsidence, independent of Poisson's ratio. Inflation of dikes, however, produces a contraction of the host rocks such that $\Delta V_{\text{uplift}}/\Delta V_{\text{injection}} = (1+2\nu)/2$, the opposite sense of host rock volumetric strain obtained from the model of a point source of uniform expansion. For the case of $\nu = 0.25$, uplift volume is 3/4 of dike injection volume.

As shown by McTigue (1987) and discussed here, the point source of uniform expansion provides an excellent approximation to the expansion of a finite spherical chamber buried at shallow depth and can be used as a reasonable 'near-field' model despite the indeterminacy of the size of the chamber. The infinitesimal sheet can be integrated to a finite size, thus providing a near-field model for inflation and deflation of sheet-like magma reservoirs. This has been performed

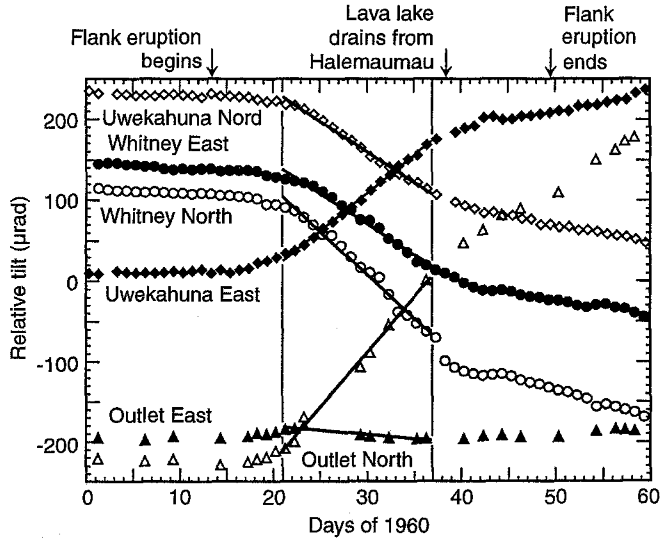


Fig. 3 East and north components of tilt measured at three vaults during the first 60 days of 1960. Vertical lines denote interval from 21 January to 5 February used by Eaton (1962) to estimate the Mogi source of deflation. Solid lines are best fits to data collected during this interval. Locations of vaults shown in Fig. 4

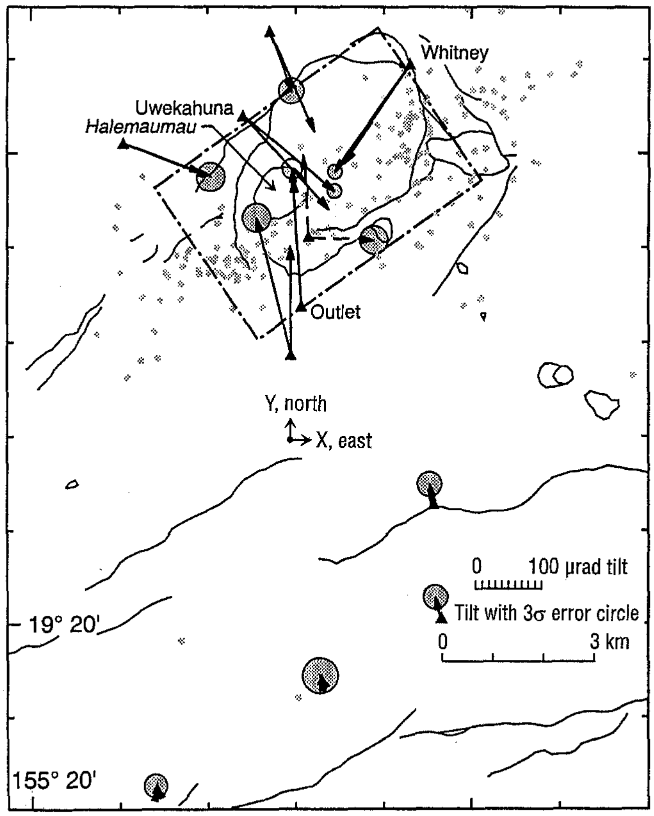


Fig. 4 Kilauea summit. Earthquake epicenters recorded between 21 January and 5 February 1960 are shown by small dots. Tilts accompanying deflation are shown by vectors with 3σ error circles. Tilt vectors without error circles are computed from a best-fit dislocation model of a rectangular horizontal sill (Table 2) striking N 65° E, the surface projection of which is also shown. Tilt at one station, shown by a dashed vector, is anomalous, as shown in this example by comparison with that predicted by the model. The origin of the coordinate system used for model calculations defines the positions of the deflation sources listed in Tables 1 and 2

for finite rectangular shapes (see Okada 1985), thus adding a length and width to the source parameters. It follows from the integration that the ratio of uplift volume to injection volume is also described by Eq. (10) and we have confirmed this with numerical calculations. (The injection volume for an opening dislocation is simply the product of the length, width and displacement; in contrast with some applications of Mogi's model, we are unaware of any workers who have calculated the volume of surface uplift as an approximation to the volume of magma accumulation in a rectangular dislocation.)

The 1960 collapse of Kilauea Crater

We now re-examine water-tube tilt data collected during the 1960 lower east rift zone eruption of Kilauea, which began on 13 January (Eaton 1962). The accompanying collapse of the summit, the largest since 1924, started 4 days later and continued well after the eruption ended on 18 February (Fig. 3). Measurements were made at seven stations between 19 and 26 January and then repeated between 4 and 6 February. Daily or near-daily measurements were also made at three other stations and these show that the summit was subsiding at a nearly constant rate, although subsidence began earlier and continued for some time thereafter. There are two orthogonal components of tilt at each station and so we can obtain a total of 20 tilt component estimates at 10 stations. To account for the non-coincidence of the measurements during the deflation, we first calculate tilt rates by least-square fits of the measurements to straight lines and then use these to estimate the tilts accumulated during the 15 days between 21 January and 5 February, the dates cited by Eaton (1962). These tilts (Fig. 4) are directed generally towards the south end of Kilauea's caldera, the region of inferred maximum subsidence, and range up to more than 200 μradians.

The tilt at one station (shown with a dashed arrow in Fig. 4) is anomalous in the sense that it is poorly fitted to all of the deformation models discussed in the following. Inclusion of that tilt increased sum of square model residuals by 35–45%, and so we prefer a smaller ($n=18$) data set to the higher residuals. The tilt at this station, however, is too large to be spurious. More likely, it was produced by a localized source of motion very different from that which caused the subsidence. The station with the anomalous behavior is near the crater Halemaumau, where Eaton (1962) suggested that growth of an unseen fissure caused draining, on 7 February, of lava ponded there and cooling since 1952. This produced a 50 m collapse of the crater floor. The crater is about 1 km in diameter, so the volume of draining may have been as much as about 40×10^6 m^3, which is substantial compared with the withdrawal volumes estimated below, as well as the 120×10^6 m^3 erupted volume estimated by Eaton (1962).

We also show (Fig. 4) 199 earthquake epicenters, timed from a local array of four stations, for the same 15 day period. The events were relocated using the methods and velocity model described by Klein (1978, 1981). Of those events, about 75% were fairly shallow, less than 2 km; most epicenters are within a zone, 8–10 km long and 2–4 km wide, trending N65°E from the upper south-west rift zone across the south caldera to the north-east of the caldera. Only two of the earthquakes exceeded magnitude 3. The zone of earthquakes traverses the site of the 5-week long Kilauea Iki eruption that ended on 21 December 1959.

Okamura (1988) reports an average closure error of 0.014 mm for the portable water-tube instruments, corresponding to a reading precision of 0.6 μradian. Seven of these 50 m base stations were read twice during the deflation. Although the closure error of the instruments permanently installed in the three concrete vaults is better than that of the portable instrument, the 3 m base is considerably less than 50 m. The precision of measurements at the three vaults is taken therefore to be the same as that at the seven stations. These estimates of precision do not correspond to accuracy, which includes errors introduced by the imperfect coupling of the stations to a representative volume of the nearby ground surface. In leveling, this source of error is referred to as benchmark instability and can be measured in tenths of millimeters (Savage et al. 1979; Delaney et al. 1993, 1994). To obtain an estimate of the accuracy, we summed the residuals obtained for linear fits to the data collected at the three vaults between 21 January and 5 February (Fig. 3) and found an average error that was 7.75 times greater than the precision, or 4.65 μradian, which was applied to all measurements. Final error estimates also depended on the number and timing of measurements at each of the stations and are displayed as 3σ error circles in Fig. 4. Insofar as this error includes the non-constant rate of subsidence, it is probably larger than the actual error associated with measurement of the tilt at each station.

Analysis of the 1960 tilt data

We fitted the tilt data to five elastic deformation models, using $\nu=0.25$. The first is the Mogi model. The second is a horizontal, or sill-like, infinitesimal sheet. Both models have $m=4$ model parameters, the maximum surface displacement, which is positioned directly above the source, the depth of that source, and its east and north positions relative to an arbitrary position defining the origin of the model coordinate system, N19°21.99′ latitude and W155°17.09′ longitude (Fig. 4). The remaining models are motivated by the north-easterly trending zone of co-deflationary seismicity, which we feel to be inconsistent with any point or infinitesimal sheet model. We therefore examine models of deflation from sheet-like chambers capable of attaining elongate form. In the third model, the sheet is constrained to be horizontal and to strike parallel to the zone of seismicity, adding a length and width to obtain $m=6$ model parameters. The fourth model allows the sheet to have a non-zero dip, $m=7$, and the fifth model allows it to vary in strike and dip, $m=8$.

The best-fit models were first obtained by minimizing the sum of square difference between the model predictions $\hat{y}$ and observations Y of the tilts, $\hat{\chi}^2=\Sigma_{i=1}^n (\hat{y}_i-Y_i)^2$ (Table 1). We repeated the calculations incorporating the estimates of errors σ_i discussed above by minimizing the normalized sum of square difference between the predictions y and observations, $\chi^2=\Sigma_{i=1}^n (y_i-Y_i)^2/\sigma_i^2$ (Table 2). Model parameters obtained from weighted and unweighted inversions differ in a fashion that is not consistent from one model to the next. Estimates of injection volume, for instance, are as much as 25% larger or smaller when error estimates are incorporated.

Table 1 Results for 1960 Kilauea summit collapse. Tilt data not weighted by errors ($n=18$)

		Mogi	Horizontal point of opening	Horizontal sill azimuth 55°	Sill azimuth 55°	Sill
m		4	4	6	7	8
$\hat{\chi}^2$	(μrad^2)	14732	14556	8746	5837	4911
X_{center}	(km)	0.32	0.31	0.53	0.89	1.72
Y_{center}	(km)	4.70	4.72	4.53	4.38	3.77
Depth	(m)	4.12	5.59	3.13	3.90	4.65
U_{max}	(m)	−0.93	−0.89			
U_3	(m)			−1.42	−2.01	−46.9
Dip	(°NW)				20.6	36.1
Azimuth	(°)					37.9
Length	(km)			5.66	5.80	5.15
Width	(km)			3.61	3.87	0.27
$\Delta V_{injection}$	(10^6 m^3)	−66.4	−58.3	−29.0	−45.1	−67.7
ΔV_{uplift}	(10^6 m^3)	−99.6	−58.3	−29.0	−43.7	−59.1

Table 2 Results for 1960 Kilauea summit collapse. Tilt data weighted by errors ($n=18$)

		Mogi	Horizontal point of opening	Horizontal sill azimuth 55°	Sill azimuth 55°	Sill
m		4	4	6	7	8
χ^2	(μrad^2)	442	429	323	195	136
X_{center}	(km)	0.47	0.47	0.60	0.85	1.78
Y_{center}	(km)	4.97	4.95	4.96	4.73	3.92
Depth	(m)	4.42	5.59	3.72	3.16	4.17
U_{max}	(m)	−0.98	−0.90			
U_3	(m)			−1.82	−1.36	−8.99
Dip	(°NW)				15.8	37.0
Azimuth	(°)					33.5
Length	(km)			5.44	5.69	5.34
Width	(km)			3.63	4.36	1.15
$\Delta V_{injection}$	(10^6 m^3)	−80.3	−58.9	−36.0	−33.6	−55.3
ΔV_{uplift}	(10^6 m^3)	−120.4	−58.9	−36.0	−33.0	−50.3

There are important advantages gained by weighting data with even approximate estimates of their accompanying errors. A hypothetical perfect model would have a normalized sum of square residual such that $\chi^2 \simeq n-m$. A glance at Table 2 shows that $\chi^2 \gg n-m$ for all models, indicating that they fail to account for all of the tilting. An important conclusion, therefore, is that processes other than the deflation parameterized by the single, idealized magma chamber models contribute substantially to the tilts. Alternatively, it is possible that the estimated measurement error, 4.65 μradian, is too small. This seems unlikely; it would need to be three to six times larger for the models to account for all of the remaining signal.

We now compare the dislocation models themselves using the results shown in Table 2. The maximum surface displacements and positions of the 'deflation center' estimated by the Mogi and infinitesimal sill models are virtuary identical, even though the source depths differ by 1 km. The positions of the centers of the rectangular sheets estimated from the remaining three models varies through 1.5 km, generally east and south of the deflation center determined from the Mogi model. Maximum surface uplifts or subsidences are skewed from the center of non-horizontal sheets. The estimated position of the center of the horizontal sheet is within 150 m of that of the point source models, suggesting that rectangular sheet models can provide a well-defined deflation center consistent with that traditionally obtained from the Mogi model. The rectangular sheet models yield depths of 3–4 km and are all more shallow than the point source models. Although all rectangular sheet models have lengths of about 5.5 km, the widths range between 1.2 and 4.4 km. The model with the narrowest sheet also has the largest deflation, 9 m, and a 37° dip, both of which seem physically unrealistic. The remaining two rectangular sheet models have deflations of 1.8 m and 1.4 m; the remaining model that allows for non-horizontality yields a dip of only 16°.

Withdrawal volumes estimated from the model parameters vary greatly (Table 2). The largest estimate, 80×10^6 m^3, comes from the Mogi model and the smallest, 34×10^6 m^3, comes from the model of a non-horizontal rectangular sheet dipping 16° to the north-west and striking parallel to the zone of earthquakes. Estimates of the volume of surface deflation vary even more than withdrawal volumes, with the Mogi model predicting a value 3.5 times larger than that for the sheet with the smallest subsidence volume. In any event, the surface deflation volume estimated from the Mogi model is substantially larger than any other estimated volume.

All of the residuals for the rectangular sheet models are smaller than those for the Mogi and infinitesimal sill models. This is to be expected as a consequence of the increased number of model parameters m. No model, however, is statistically superior to the others at the 95% confidence level. The rectangular sheet models do have the advantage of spanning the zone of co-deflationary seismicity (Fig. 4). Also, the sheet models are shallower than the others, which is consistent with the dominant 0–2 km hypocentral depths. Noting that the model estimates themselves have errors, which we have not estimated, that propagate from the measurement errors, we conclude that the withdrawal volume of magma is poorly constrained. For the 15 days of the 1960 collapse between 21 January and 5 February, we prefer an estimate of the withdrawal volume drawn from the sheet models, $30\text{–}40 \times 10^6$ m^3. Any estimate within the range $20\text{–}90 \times 10^6$ m^3, however, cannot be rejected.

Discussion

Many of the assumptions used to obtain the results presented here are, of course, questionable. Most rocks have neither homogeneous nor isotropic elastic properties. Rocks near magma reservoirs are subject to high temperatures and presumed inelastic deformation (see Davis et al. 1974). Rocks near the surface are typically poorly lithified and possess open fractures, lava tubes and other inhomogeneities. In the case of Mogi's result, dilation is greatest near the ground surface above the magma reservoir (Fig. 2), and so it is this area where the assumptions used to obtain solutions may be most critical. Moreover, magma reservoir geometries are almost certainly much more complicated than any model would suggest. Dieterich and Decker (1975), who developed two-dimensional numerical solutions for expanding magma reservoirs, showed that height and tilt data would be unable to distinguish a circular chamber, a vertically elongate oval chamber and a sill arranged such that their depths to center vary through a factor of about three. (They also showed that the incorporation of measurements of horizontal motion would, ideally, be able to distinguish among these models.) Finally, all methods presently available to invert ground surface displacement data use sources of deformation that are not necessarily physically plausible; deformations are caused by pressures, generally non-uniformly distributed, applied by magma and not by *ad hoc* specifications of the displacements they produce on the walls of the magma reservoir.

Dzurisin et al. (1980) discuss factors, such as filling and emptying of voids and microcracks, that would be reflected neither in the volume of surface displacement nor in a calculated volume of magma accumulation or withdrawal obtained from fits of geodetic data to Mogi's model. Dvorak et al. (1983) suggest that, in view of these factors, the volume of uplift is a minimum estimate of the volume of magma accumulation. The injection volume, Equation (7), provides a lesser and more robust estimate. Dvorak and Dzurisin (1993), fitting geodetic data to Mogi's model, found an apparent consistency between volumes of subsidence and corresponding volumes of lava erupted between 1969 and 1983 at Kilauea Volcano, offering some confirmation of Eaton's (1962) result. In view of the 3/2 difference between uplift volume and injection volume implicit in Mogi's model, their assertion that the two are equal poses an inconsistency in the use of that model. This inconsistency could be removed by treating the host rock as incompressible, although there is little justification for this approximation.

If the equality of uplift and injection volumes is to be asserted, then a model of a sill-like body, or any other that produces no net change in the volume of the host rocks, is more appropriate than a Mogi-type model. In fact, the vertical displacement of the ground surface above an infinitesimal plane of opening from a horizontal surface resembles that of the point of uniform dilation

$$u_z(\rho) = u_z^{\max}[1+(p/d)^2]^{-5/2} \tag{11}$$

where

$$u_z^{\max} = \frac{3\Delta V_{\text{injection}}}{2\pi d^2} \tag{12}$$

as simplified from Okada [1985, Equation (10)]. The area of the dislocation cannot be distinguished from the magnitude of opening on that surface. Equations (11) and (12) are a valid approximation if the dimensions of the surface are small compared with depth *d*. Comparing Eqs. (2) and (11), inflation of a small sill-like chamber produces a narrower uplift than the inflation of a spherical chamber at the same depth. Each of the point source models accounts for the 1960 collapse data equally well (Table 2), having similar estimates of maximum subsidence, 0.98 and 0.90 m, respectively. The estimated injection and uplift volumes, on the other hand, differ by factors of 36 and 105%, respectively.

There are large uncertainties in the withdrawal and subsidence volumes determined from the 1960 tilt data and in the determination of the volume of 1960 lava accumulated in the heavy vegetation and dispersed after flowing into the ocean. Moreover, the tilts discussed here and by Eaton (1962) were measured during an interval much shorter than the duration of summit deflation (Fig. 3). In view of these factors, as well as the large volume of magma lost from the summit lava lake in February 1960 and the 50–60 km distance from the summit to the eruptive vents, we do not feel that there exists a valid basis for a comparison of the volumes of magma withdrawn from the summit reservoir with lava erupted along the lower east rift zone that would justify use of the latter to constrain the former.

We are attracted to a model of deflation of a sill-like chamber to account for the 1960 collapse at Kilauea. The possibility that sill-like chambers underlie Hawaiian volcanoes was pursued by Ryan et al. (1983), who examined deflation events at Kilauea but did not compare the results with those obtained from other models, especially Mogi's. The analysis of Ryan et al. (1983) was well motivated by the observation that intrusive sheets are commonly exposed in eroded sections of Hawaiian volcanoes. Although dikes are far more common than sill-like bodies, acccumulation in and withdrawal from sills can well account for caldera-wide uplift and subsidence where that due to dikes cannot. Moreover, that subsidence near the summit persisted well after the flank eruption ended in 1960 (Fig. 3) offers a reminder that hydraulics and storage at locations intermediate between the summit and the eruption site also exert some control on the overall flux of magma.

Some magma reservoir parameters, such as size, position and depth, should perhaps remain relatively constant from one event to the next. These parameters, then, might be determined globally across all data sets. In this fashion, more data would be available to characterize better the processes causing inflation, deflation and associated faulting during particular intervals. Such a strategy was pursued by Yang et al. (1991), who found that apparent migration of the center of ground surface uplift at Kilauea Volcano could be explained by coincidence of the magma reservoir inflation with the growth of nearby dikes.

Our primary interest in examining the 1960 data is to document the poor agreement of withdrawal volumes estimated from the various models and from the slightly different methods that ignore or treat the measurement errors (Tables 1 and 2), as well as to exemplify the differences between withdrawal and subsidence volumes. It can be argued that more and better data, as is now easily collected, would produce better results in the sense that the most realistic deformation model could be more readily identified. This is not entirely so. The 1960 tilts are much larger than the accuracies of the measurements, so that even more accurate data would not, in itself, improve the results. More data, even of lesser accuracy, collected at other, more widely dispersed stations, however, would certainly improve the resolution of the source of ground surface motion.

Conclusions

In summary, all rocks are compressible and volume changes in the host rocks surrounding a magma reservoir can comprise a significant fraction of the volume of ground surface uplift or subsidence caused by magma accumulation or withdrawal. Unfortunately, both the sense and magnitude of host rock volume change depends on the resevoir geometry. Where the volume of surface uplift or subsidence is known from a model, such as Mogi's, the volume of magma transfer should be calculated from the source parameters. The assumption that the uplift or subsidence volume is a direct measure of injection or withdrawal volume should be avoided. Also, various models for magma chamber inflation and deflation can offer very different estimates of volume change when applied to the same data. Unless one of these models is clearly superior to the others, and we suspect that this is often problematic, the uncertainty in the volumes of magma injection or withdrawal is best estimated from the range of model results.

Acknowledgements Jerry Eaton, Roger Denlinger and Paul Segall provided useful information and discussions before and during the preparation of this paper. Roger Denlinger, Jim Dieterich, Paul Davis and Akira Takada each supplied especially instructive reviews and we thank them for their efforts.

References

Davis PM, Hastie LM, Stacey FD (1974) Stresses within an active volcano – with particular reference to Kilauea. Tectonophysics 22:355–362

Delaney PT, Miklius A, Arnadottir T, Okamura AT, Sako MK (1993) Motion of Kilauea volcano during sustained eruption from the Puu Oo and Kupaianaha vents, 1983–1991. J Geophys Res 98:17801–17820

Delaney PT, Miklius A, Arnadottir T, Okamura AT, Sako MK (1994) Motion of Kilauea volcano during sustained eruption from the Puu Oo and Kupaianaha vents, 1983–1991: supplemental information. US Geol Surv Open File Rep 94–567

Dieterich JH, Decker RW (1975) Finite element modeling of surface deformation associated with volcanism. J Geophys Res 80:4094–4102

Dvorak JJ, Dzurisin D (1993) Variations in magma supply rate at Kilauea Volcano Hawaii. J Geophys Res 98:22255–22268

Dvorak JJ, Okamura AT, Dieterich JH (1983) Analysis of surface deformation data, Kilauea Volcano, Hawaii: October 1966 to September 1970. J Geophys Res 88:9295–9304

Dzurisin D, Anderson LA, Eaton GP, Koyanagi RY, Lipman PW, Lockwood JP, Okamura RT, Puniwai GS, Sako MK, Yamashita KM (1980) Geophysical observations of Kilauea volcano, Hawaii, 2. Constraints on the magma supply during November 1975–September 1977. J Volcanol Geotherm Res 7:241–269

Eaton JP (1962) Crustal structure and volcanism in Hawaii. Am Geophys Union Geophys Monogr 6:13–29

Jackson DB, Swanson DA , Koyanagi RY, Wright TL (1975) The August and October 1968 east rift eruptions of Kilauea Volcano, Hawaii. US Geol Surv Prof Pap 890

Klein FW (1978) Hypocenter location program HYPOINVERSE. US Geol Surv Open File Rep 78–694

Klein FW (1981) A linear gradient crustal model for south Hawaii. Bull Seism Soc Am 71:1503–1510

McTigue DF (1987) Elastic stress and deformation near a finite spherical magma body: resolution of the point source paradox. J Geophys Res 92:12931–12940

Mogi K (1958) Relations between the eruptions of various volcanoes and the deformations of the ground surfaces around them. Bull Earthquake Res Inst, Univ Tokyo 36:99–134

Okada Y (1985) Surface deformation due to shear and tensile faults in a half-space. Bull Seism Soc Am 75:1135–1154

Okamura AT (1988) Water-tube and spirit-level tilt data, Hawaiian Volcano Observatory 1958–1986. US Geol Survey Open File Rep 88–237

Rundle JB, Whitcomb JH (1984) A model for deformation in Long Valley, California, 1980–1983. J Geophys Res 89:9371–9380

Ryan MP, Blevins JYK, Okamura AT, Koyanagi RY (1983) Magma reservoir mechanics: theoretical summary and application to Kilauea Volcano. J Geophys Res 88:4147–4182

Savage JC, Clark MM (1982) Magmatic resurgence in Long Valley caldera, California: possible cause of the 1980 Mammoth Lakes earthquakes. Science 217:531–533

Savage JC, Prescott WH, Chamberlain JF, Lisowski M, Mortensen CE (1979) Geodetic tilt measurements along the San Andreas fault in central California. Bull Seism Soc Am 69:1965–1981

Touloukian YS, Judd WR, Roy RF (1981) Physical Properties of Rocks and Minerals. McGraw-Hill, New York, 548 pp

Yang X, Davis PM, Delaney PT, Okamura AT (1992) Geodetic analysis of dike intrusion and motion of the magma reservoir beneath the summit of Kilauea Volcano, Hawaii: 1970–1985. J Geophys Res 97:3305–3324

Editorial responsibility: W. Hildreth